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Conformity between linear response and binary collision treatments of an ion energy loss in a magnetized quantum plasma

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Abstract. In this paper the energy loss of a heavy ion moving in a magnetized quantum electron plasma is considered within the linear response and binary collision treatments. Treating the electron-ion interaction force as a small perturbation to the electron nth Landau level we show within the second order perturbation theory the conformity between these two models.

PACS. 52.40.Mj Particle beam interactions in plasmas – 34.50.Bw Energy loss and stopping power – 03.65.Nk Scattering theory – 52.20.Hv Atomic, molecular, ion, and heavy-particle collisions

1 Introduction

The electron collisions with ions, the ion energy loss and related processes in a strongly magnetized plasma are one of the main subject of recent theoretical and experimental investigations (see, e.g., Refs. [1–18]). These processes are the basis of the transport phenomena (see, e.g., [1] and references therein), plasma heating and magnetic confinement of thermonuclear plasmas. In addition, the electron-ion collisions and the related processes in a magnetized plasma play an important role in the cooling of heavy-ion and antiproton beams by electrons (or positrons) [2] and in the energy transfer for heavy-ion inertial fusion [3].

For a theoretical description of the energy loss of ions in a plasma there exist two standard approaches. The dielectric linear response (LR) treatment considers the ion as a perturbation of the target plasma and the stopping is caused by the polarization of the surrounding medium. Within the LR treatment the stopping power can receive a dynamic contribution from collective plasma excitations. It requires a cutoff at small distances where hard collisions between ion and electrons cannot be treated any more as a weak perturbation. This topic has been intensively investigated and since 1960s, a number of theoretical calculations of the stopping power within LR treatment in a magnetized plasma have been presented both within classical (see, e.g., [4–8] and references therein) and quantum mechanical formalisms [9].

Within the second approach the stopping power is calculated as the result of the energy transfers in successive binary collisions (BC) between the ion and the electrons. Here it is essential to consider appropriate approximations for the screening of the Coulomb potential by the plasma or an effective upper cutoff for the impact parameters, to account for screening.

As the problem of two charged particles colliding in an external magnetic field cannot be solved in closed form a number of levels of approximations have been developed. Numerical and analytical calculations have been performed for classical BC between magnetized electrons [10,11] and for collisions between magnetized electrons and ions [12–17]. In the later case as an ion is much heavier than an electron, its uniform motion is only weakly perturbed by collisions with the electrons. Hence for the electron-ion collision the perturbation theory in the ion charge Z for small angle scattering might provide an useful information. This has been done previously in first order in Z and for an ion at rest [14] as well as in second order for uniformly moving ion [15,16]. The large angle scattering in the strong magnetic field shows an chaotic behavior of the scattering events for low energy electrons [17] where the scattering angle may have a fractal dependence on the impact parameter.

Both treatments, LR and BC, can be regarded as complementary to each other. In particular, the lower cutoff

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required within LR treatment should be provided by the BC. In this paper within the quantum mechanical formalism we consider the Coulomb interaction with the ion as a perturbation to the Landau states of the magnetized electrons while the ion motion describes classically and remains unchanged (heavy ion). Starting with the BC treatment we show the full agreement with the LR result of the stopping power. Previously this has been done within classical approach [16].

2 Linear response formulation

We recall briefly the main results of the LR theory for the ion energy loss in a magnetized quantum plasma. The external constant magnetic field is considered to be parallel to the z-direction $\mathbf{B} = B\mathbf{e}_z$ and $|\mathbf{e}_z| = 1$. The plasma is specified by its temperature T and by the plasma frequency $\omega_p = \left(n_0 e^2/m\varepsilon_0\right)^{1/2}$, where n_0 is the electron density. The influence of the plasma ions on the projectile ion energy loss is neglected. The motion of the electrons are characterized by the cyclotron frequency $\omega_c = eB/m$ or the magnetic length $\lambda_B = (\hbar/m\omega_c)^{1/2}$, where m is the electron mass.

Consider a test heavy ion of mass M and charge Ze ($-e$ is the electron charge) that moves with velocity $\mathbf{v}_i$ in a magnetized plasma. We assume a mass of the ion $M \gg m$ such that a classical description of its motion with rectilinear trajectory is applicable. With that for the stopping power (SP) we obtain (see, e.g., [4–9])

$$S_{\rm LR} = -\frac{dE_i}{ds} = \frac{Z^2 e^2}{\varepsilon_0 v_i (2\pi)^3} \int d\mathbf{k} \frac{\mathbf{k}\cdot\mathbf{v}_i}{k^2} {\rm Im} \frac{-1}{\varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v}_i)}. \quad (1)$$

The dielectric function $\varepsilon(\mathbf{k}, \omega)$ of a homogeneous plasma is given by $\varepsilon(\mathbf{k}, \omega) = 1 + V(\mathbf{k})\chi^{(0)}(\mathbf{k}, \omega)$, where $V(\mathbf{k})$ is the Fourier transformed electron-electron interaction potential $V(\mathbf{k}) = (2\pi)^{-3} e^2/(\varepsilon_0 k^2)$. $\chi^{(0)}(\mathbf{k}, \omega)$ is the susceptibility of magnetized quantum plasma (see, e.g., [9])

$$\chi^{(0)}(\mathbf{k}, \omega) = \frac{2\pi}{\lambda_B^2} \sum_{\sigma=\pm 1/2} \sum_{n;n'=0}^{\infty} F_{nn'}^2(\zeta) \times \int_{-\infty}^{\infty} dq_z \frac{f\left(E_{n\sigma}(q_z)\right) - f\left(E_{n'\sigma}(q_z + k_z)\right)}{E_{n'\sigma}(q_z + k_z) - E_{n\sigma}(q_z) - \hbar\omega - i0}, \quad (2)$$

where $\zeta = k_\perp^2 \lambda_B^2/2$. $k_\perp$ denotes the component of $\mathbf{k}$ perpendicular to the external magnetic field. The positive infinitesimal $+i0$ in equation (2) guarantees the causality of the response. Here the summation is carried out over all Landau levels $n, n' = 0, 1, 2, ...$ and spin variable $\sigma = \pm 1/2$. The arguments of the Fermi-Dirac function $f(E)$ are given by the eigenvalues of the free particles:

$$E_{n\sigma}(q_z) = \frac{\hbar^2 q_z^2}{2m} + \hbar\omega_c \left(n + \sigma + \frac{1}{2}\right). \quad (3)$$

The function $F_{nn'}(\zeta)$ is given by [19]

$$F_{nn'}(\zeta) = \left(\frac{n!}{n'!}\right)^{1/2} \zeta^{(n'-n)/2} e^{-\zeta/2} L_n^{n'-n}(\zeta) \ \ (n \leq n'), \quad (4)$$

$F_{nn'}(\zeta) = (-1)^{n-n'} F_{n'n}(\zeta)$ $(n > n')$, and $L_n^{n'}(\zeta)$ are the generalized Laguerre polynomial [20].

The relation between the chemical potential μ and the electron density n_0 is established by the normalization rule

$$\frac{1}{(2\pi\lambda_B)^2} \sum_{n\sigma} \int_{-\infty}^{\infty} dq_z f\left(E_{n\sigma}(q_z)\right) = n_0. \quad (5)$$

In the classical description the SP is derived from linearized Vlasov equation, where the self-consistent electrostatic potential is determined by Poisson's equation. Within this approach the dielectric function in equation (1) is given by its classical representation (see, e.g., [21]). As stated above a lower cutoff should be introduced here to avoid the divergence of the $\mathbf{k}$ integral at small distances. In contrast to that, in quantum mechanical description the wave nature of the electrons leads to a quantum expression (2) for the dielectric function and avoids the cutoff procedure at small distances.

In general the collective excitations (i.e. magnetized plasma modes) contribute to the stopping power and these contributions are contained in $\varepsilon(\mathbf{k}, \mathbf{k}\cdot\mathbf{v}_i)$ in equation (1). In the following we assume that

$${\rm Im} \frac{-1}{\varepsilon(\mathbf{k}, \omega)} = \frac{{\rm Im}\varepsilon(\mathbf{k}, \omega)}{|\varepsilon(\mathbf{k}, \omega)|^2} \simeq \frac{{\rm Im}\varepsilon(\mathbf{k}, \omega)}{|\varepsilon(\mathbf{k}, 0)|^2} \quad (6)$$

which means that the stopping power does not receive any contribution from the dynamic collective plasma modes. However the static collective contributions (i.e. screening) can be easily reintroduced by replacing the ion bare Coulomb potential, $\Phi_0(\mathbf{k}) = (2\pi)^{-3} Ze/\left(\varepsilon_0 k^2\right)$, with an shielded one $\Phi_{\rm ie}(\mathbf{k}) = \Phi_0(\mathbf{k})\varepsilon^{-1}(\mathbf{k}, 0)$. Then equation (6) amounts to neglecting the excitation of the collective modes but accounts the static shielding of the ion.

In this simplified LR approximation the stopping power thus reads

$$S'_{\rm LR} = \frac{e^2}{v_i} \int d\mathbf{k} \, |\Phi_{\rm ie}(\mathbf{k})|^2 \, (\mathbf{k}\cdot\mathbf{v}_i) \, {\rm Im}\chi^{(0)}(\mathbf{k}, \mathbf{k}\cdot\mathbf{v}_i). \quad (7)$$

3 Binary collision formulation

Consider now the electron-ion BC in the presence of quantizing magnetic field. We consider an electron and projectile heavy ion moving in a homogeneous magnetic field $\mathbf{B}$. We assume that the particles interact with the potential $-e\Phi_{\rm ie}(\mathbf{r} - \mathbf{v}_i t)$, where $\mathbf{r}$ and $\mathbf{v}_i t$ are the coordinates of colliding particles. For charged particles the function $\Phi_{\rm ie}(\mathbf{r})$ can be expressed, for instance, by the Yukawa type screened potential, $\Phi_{\rm ie}(\mathbf{r}) = Ze\exp(-r/\lambda)/4\pi\varepsilon_0 r$ (λ is the screening length), for application in plasmas.

Our starting point is the Schrödinger equation

$$\left(\hat{H}_0 + \hat{H}_1(t)\right) \psi = i\hbar \frac{\partial \psi}{\partial t} \quad (8)$$

with the time-dependent Hamiltonian $\hat{H}_1(t) = -e\Phi_{\rm ie}(\mathbf{r} - \mathbf{v}_i t)$ and the Hamiltonian of free electron

$$\hat{H}_0 = \frac{1}{2m}\left(\hat{\mathbf{p}} + e\mathbf{A}\right)^2 + \hbar\omega_c \hat{s}_z. \quad (9)$$

Here $\mathbf{A}$ is the vector potential of the magnetic field with the components $A_x = A_z = 0$ and $A_y = Bx$. $\hat{s}_z$ is the spin operator. We note that due to it commutes with the full Hamiltonian in equation (8), and the coefficient of $\hat{s}_z$ in equation (9) is a constant, $\hat{s}_z$ is conserved and the spin and coordinate variables in equation (8) are separable.

Alternatively, one can consider the scattering process with the stationary interaction Hamiltonian $\hat{H}_1 \sim \Phi_{\rm ie}(\mathbf{r})$, where $\mathbf{r}$ is the relative radius vector of the particles. For instance, the similar program was carried out in reference [18]. This leads to the energy conservation of the particles. In contrast to the previous works [18] we use here more realistic time-dependent Hamiltonian $\hat{H}(t) = \hat{H}_0 + \hat{H}_1(t)$. Thus, after interaction the energy of an electron has not certain value since its energy does not conserve.

We seek an approximate solution of equation (8) in which the interaction potential is considered as a perturbation. We start with the zero-order unperturbed eigenstates which are described by the zero-order Schrödinger equation

$$\hat{H}_0 \psi_\alpha^{(0)} = i\hbar \frac{\partial \psi_\alpha^{(0)}}{\partial t}, \quad (10)$$

where the corresponding eigenstates are labeled by $\alpha = \{n, \sigma, q_y, q_z\}$. Here $\psi_\alpha^{(0)}$ is the unperturbed electron wave function in the state α which is given by $\psi_\alpha^{(0)}(\mathbf{r}, t) = \psi_\alpha^{(0)}(\mathbf{r}) e^{-i\Omega_\alpha t}$ [22]

$$\psi_\alpha^{(0)}(\mathbf{r}) = \frac{A_n}{2\pi \lambda_B^{1/2}} e^{i(q_y y + q_z z)} e^{-\xi^2/2} H_n(\xi) u_\sigma \quad (11)$$

with $\Omega_\alpha = E_\alpha/\hbar$, $\xi = x/\lambda_B + q_y \lambda_B$, $A_n = (\sqrt{\pi} 2^n n!)^{-1/2}$. In equation (11) u_σ is the spin wave function, H_n is the Hermite polynomial, and E_α is given by equation (3).

We now seek the solution of the perturbed equation (8) in the form

$$\psi_\alpha(\mathbf{r}, t) = \sum_\beta a_{\alpha\beta}(t) \psi_\beta^{(0)}(\mathbf{r}, t), \quad (12)$$

where the expansion coefficients are function of time and $\beta = \{n', \sigma', q_y', q_z'\}$. Substituting (12) into equation (8), and recalling that the function $\psi_\beta^{(0)}(\mathbf{r}, t)$ satisfy equation (10) we obtain

$$\dot{a}_{\alpha\beta}(t) = -\frac{i}{\hbar} \sum_\gamma h_{\beta\gamma}(t) a_{\alpha\gamma}(t), \quad (13)$$

where

$$h_{\beta\gamma}(t) = e^{i\Omega_{\beta\gamma} t} \int d\mathbf{r} \psi_\beta^{(0)*}(\mathbf{r}) \hat{H}_1(t) \psi_\gamma^{(0)}(\mathbf{r}) \quad (14)$$

are the matrix elements of the perturbation, including the time factor. Here $\hbar\Omega_{\beta\gamma} = E_\beta - E_\gamma$.

The expression (13) is exact equation for the expansion coefficients $a_{\alpha\beta}(t)$. Since in equation (12) as the unperturbed wave function we take the wave function of the αth stationary state, in the zero order (in the absence of particles interaction) we find $a_{\alpha\beta}^{(0)}(t) = \delta_{\alpha\beta}$. The equation for the first-order correction is then given by $\dot{a}_{\alpha\beta}^{(1)}(t) = -(i/\hbar) h_{\beta\alpha}(t)$. Assuming that all corrections vanish at $t \to -\infty$ and writing the ion-electron interaction potential using Fourier transformation in space from equation (12) for the first-order electron wave function we find

$$\psi_\alpha^{(1)}(\mathbf{r}, t) = \frac{e}{\hbar} \int d\mathbf{k} \Phi_{\rm ie}(\mathbf{k}) \sum_\beta \psi_\beta^{(0)}(\mathbf{r}, t) \times S_{\beta\alpha}(\mathbf{k}) \frac{e^{i(\Omega_{\beta\alpha} - \omega)t}}{\Omega_{\beta\alpha} - \omega - i0}, \quad (15)$$

where $\omega = \mathbf{k} \cdot \mathbf{v}_i$, and the matrix $S_{\beta\alpha}(\mathbf{k})$ is given by [19]

$$S_{\beta\alpha}(\mathbf{k}) = \int d\mathbf{r} \psi_\beta^{(0)*}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} \psi_\alpha^{(0)}(\mathbf{r}) = \delta_{\sigma\sigma'} \delta_{k_y + q_y; q_y'} \times \delta_{k_z + q_z; q_z'} e^{-ik_x \lambda_B^2 (q_y + k_y/2)} \times \exp\left[i\left(n' - n\right) \arctan \frac{k_x}{k_y}\right] F_{nn'}(\zeta). \quad (16)$$

At large $t \to \infty$ the interaction force between the particles vanishes. Thus after interaction the electron at large t is described by the superposition of the stationary zero-order wave functions with the constant expansion coefficients. The first-order correction to the stationary state $\psi_\alpha^{(0)}$ can be found from equation (15). At large t from equation (15) we find $\psi_\alpha^{(1)}(\mathbf{r}, t) = \sum_\beta N_{\alpha\beta} \psi_\beta^{(0)}(\mathbf{r}, t)$, where

$$N_{\alpha\beta} = a_{\alpha\beta}^{(1)}(\infty) = \frac{2\pi i e}{\hbar} \int d\mathbf{k} \Phi_{\rm ie}(\mathbf{k}) S_{\beta\alpha}(\mathbf{k}) \delta\left(\omega - \Omega_{\beta\alpha}\right). \quad (17)$$

The quantities $N_{\alpha\beta}$ give the first order correction to the eigenstates $\psi_\alpha^{(0)}(\mathbf{r}, t)$ after an electron-ion collision. We note that the diagonal elements of this matrix $N_{\alpha\alpha} \sim \Phi_{\rm ie}(0)$. Therefore for the bare Coulomb interaction potential a lower cutoff $k_{\min} = 1/r_{\max}$ must be introduced in equation (17) to avoid the divergence at small k. Here $k_{\min}$ must account for screening with $r_{\max} \sim \lambda$.

Since the energy of an electron during electron-ion interaction is not conserved the Fermi golden rule is not applicable for calculation of the energy transfer of the electron after collision with the nonstationary ion. Here for calculation of the expected energy transfer we introduce the probability current density of the electron in the state $|\alpha\rangle$ [22]

$$\mathbf{j}_\alpha(\mathbf{r}, t) = \frac{i\hbar}{2m}\left(\psi_\alpha \nabla \psi_\alpha^* - \psi_\alpha^* \nabla \psi_\alpha\right) + \frac{e\mathbf{A}}{m} |\psi_\alpha|^2 \quad (18)$$

with the corresponding probability density $\rho_\alpha = |\psi_\alpha|^2$. Then the energy change of the electron per unit time can be calculated as

$$\frac{dW_\alpha}{dt} = -\frac{e}{L^2}\int d\mathbf{r}\mathbf{j}_\alpha(\mathbf{r},t)\cdot\mathbf{E}_{\rm ext}(\mathbf{r},t), \tag{19}$$

where $\mathbf{E}_{\rm ext}(\mathbf{r},t) = -\nabla\Phi_{\rm ie}(\mathbf{r}-\mathbf{v}_i t)$ is the electrical field created by a moving ion. Here L is the normalization length. The energy loss of the ion per unit length (stopping power) in a homogeneous electron plasma is obtained by averaging equation (19) over the unperturbed electron distribution function

$$S_{\rm BC} = \frac{L^2}{v_i}\sum_\alpha f(E_\alpha)\frac{dW_\alpha}{dt} = \frac{1}{v_i}\int d\mathbf{r}\mathbf{J}(\mathbf{r},t)\cdot\mathbf{E}_{\rm ext}(\mathbf{r},t). \tag{20}$$

Here we have introduced the averaged electrical current and electrical charge density of the electrons

$$\mathbf{J}(\mathbf{r},t) = -e\textstyle\sum_\alpha f(E_\alpha)\,\mathbf{j}_\alpha(\mathbf{r},t), \tag{21}$$
$$\rho(\mathbf{r},t) = -e\textstyle\sum_\alpha f(E_\alpha)\,\rho_\alpha(\mathbf{r},t). \tag{22}$$

Let us note that equation (20) differs from the general definition of the stopping power (see, e.g., [23]) in terms of the density matrix. To show the identity of both treatments we consider the relation $\mathbf{J}\cdot\mathbf{E}_{\rm ext} = \Phi_{\rm ie}\nabla\cdot\mathbf{J} - \nabla\cdot(\Phi_{\rm ie}\mathbf{J})$. The last term of this equation vanishes after transformation of the volume integral into the surface one according to the Gauss theorem. Using the continuity equation for the probability current and density the first term can be rewritten as

$$\rho\frac{\partial\Phi_{\rm ie}}{\partial t} - \frac{\partial}{\partial t}(\rho\Phi_{\rm ie}) = \rho\mathbf{v}_i\mathbf{E}_{\rm ext} - \frac{\partial}{\partial t}(\rho\Phi_{\rm ie}). \tag{23}$$

The last term in equation (23) is the time derivative of the function $\rho\Phi_{\rm ie}$ which represents the density of plasma potential energy $U_{\rm pl}(\mathbf{r},t)$ in the electrical field of the external ion. However, the total potential energy $\int U_{\rm pl}(\mathbf{r},t)d\mathbf{r}$ should be a constant for homogeneous and infinity plasma as can be traced from the further consideration. Therefore this term should be omitted. Substituting of the first term of equation (23) into (20) we arrive at

$$S_{\rm BC} = \frac{\mathbf{v}_i}{v_i}\cdot\int d\mathbf{r}\rho(\mathbf{r},t)\mathbf{E}_{\rm ext}(\mathbf{r},t) = \frac{\mathbf{v}_i}{v_i}\cdot\mathbf{F}_{\rm tot}, \tag{24}$$

where $\mathbf{F}_{\rm tot}$ is the total averaged electrical force acting on the plasma. In equation (24) the charge density $\rho(\mathbf{r},t)$ represents the eigenvalues of the density matrix operator considered in literature (see, e.g., [23]) which guarantees the identity of both treatments.

3.1 First-order energy transfer

Below we evaluate the general expressions (18–21) within first and second order perturbation theory. The first order energy transfer (see Eqs. (19, 20)) is proportional to the zero-order probability current density

$$\mathbf{j}_\alpha^{(0)}(\mathbf{r},t) = \frac{i\hbar}{2m}\left(\psi_\alpha^{(0)}\nabla\psi_\alpha^{(0)*} - \psi_\alpha^{(0)*}\nabla\psi_\alpha^{(0)}\right) + \frac{e\mathbf{A}}{m}\left|\psi_\alpha^{(0)}\right|^2 \tag{25}$$

or to the zero-order probability density $\rho_\alpha^{(0)} = \left|\psi_\alpha^{(0)}\right|^2$:

$$S_{\rm BC}^{(1)} = \frac{\mathbf{v}_i}{v_i}\cdot\int d\mathbf{r}\rho^{(0)}(\mathbf{r},t)\mathbf{E}_{\rm ext}(\mathbf{r},t). \tag{26}$$

Here $\rho^{(0)}(\mathbf{r},t)$ is related to the probability density $\rho_\alpha^{(0)}(\mathbf{r},t)$ according to equation (22). Using the Fourier representation of the interaction potential equation (26) can be written as

$$S_{\rm BC}^{(1)} = \frac{2\pi ie}{\lambda_B^2 v_i}\sum_{n\sigma}\int_{-\infty}^{\infty}dq_z f\left(E_{n\sigma}(q_z)\right) \times\int d\mathbf{k}\Phi_{\rm ie}(\mathbf{k})\left(\mathbf{k}\cdot\mathbf{v}_i\right)e^{-i(\mathbf{k}\cdot\mathbf{v}_i)t}\delta(\mathbf{k}). \tag{27}$$

From equation (27) we finally find $S_{\rm BC}^{(1)} = 0$. Then as for classical description [15,16] the first-order energy transfer gives then no contribution and the ion energy loss receives contribution only from higher orders.

3.2 Second-order energy transfer

Consider now the second-order energy loss which is proportional to the first-order electrical current

$$S_{\rm BC}^{(2)} = \frac{1}{v_i}\int d\mathbf{r}\mathbf{J}^{(1)}(\mathbf{r},t)\cdot\mathbf{E}_{\rm ext}(\mathbf{r},t). \tag{28}$$

Here $\mathbf{J}^{(1)}(\mathbf{r},t)$ is related to the $\mathbf{j}_\alpha^{(1)}(\mathbf{r},t)$ according to equation (21) and the quantity $\mathbf{j}_\alpha^{(1)}(\mathbf{r},t)$ is given by

$$\mathbf{j}_\alpha^{(1)}(\mathbf{r},t) = \frac{i\hbar}{2m}\Big(\psi_\alpha^{(0)}\nabla\psi_\alpha^{(1)*} - \psi_\alpha^{(0)*}\nabla\psi_\alpha^{(1)} + \psi_\alpha^{(1)}\nabla\psi_\alpha^{(0)*} - \psi_\alpha^{(1)*}\nabla\psi_\alpha^{(0)}\Big) + \frac{e\mathbf{A}}{m}\left(\psi_\alpha^{(0)}\psi_\alpha^{(1)*} + \psi_\alpha^{(0)*}\psi_\alpha^{(1)}\right). \tag{29}$$

Using equation (15) and Fourier representation of the interaction potential $\mathbf{J}^{(1)}(\mathbf{r},t)$ can be written as

$$\mathbf{J}^{(1)}(\mathbf{r},t) = \frac{ie^2}{2m}\int d\mathbf{k}\Phi_{\rm ie}(\mathbf{k})e^{-i\omega t}\sum_{\alpha;\beta}S_{\beta\alpha}(\mathbf{k})\mathbf{P}_{\alpha\beta}(\mathbf{r}) \times\frac{f(E_\alpha) - f(E_\beta)}{\Omega_{\beta\alpha} - \omega - i0}, \tag{30}$$

where we have introduced the vector

$$\mathbf{P}_{\alpha\beta}(\mathbf{r}) = \psi_\alpha^{(0)*}(\mathbf{r})\nabla\psi_\beta^{(0)}(\mathbf{r}) - \psi_\beta^{(0)}(\mathbf{r})\nabla\psi_\alpha^{(0)*}(\mathbf{r}) + \frac{2ie\mathbf{A}}{\hbar}\psi_\beta^{(0)}(\mathbf{r})\psi_\alpha^{(0)*}(\mathbf{r}). \tag{31}$$

Substituting equation (30) into equation (28) and after some lengthy calculations one arrives at the following expression for the second-order stopping power (see Appendix for more details)

$$S_{\rm BC}^{(2)} = -\frac{ie^2}{\hbar v_i}\int d\mathbf{k}d\mathbf{k}'\Phi_{\rm ie}(\mathbf{k})\Phi_{\rm ie}^*(\mathbf{k}')e^{i(\omega'-\omega)t} \times \sum_{\alpha;\beta} S_{\beta\alpha}(\mathbf{k})S_{\beta\alpha}^*(\mathbf{k}')\frac{(E_\beta - E_\alpha)\left[f(E_\alpha) - f(E_\beta)\right]}{E_\beta - E_\alpha - \hbar\omega - i0}. \quad (32)$$

Finally using the expression

$$\int_{-\infty}^{\infty} dq_y dq_y' S_{\beta\alpha}(\mathbf{k})S_{\beta\alpha}^*(\mathbf{k}') = \frac{2\pi}{\lambda_B^2}\delta_{\sigma\sigma'}\delta_{q_z';q_z+k_z}\delta(\mathbf{k}-\mathbf{k}')F_{nn'}^2(\zeta) \quad (33)$$

we obtain $S_{\rm BC}^{(2)} = S_{\rm LR}'$ (see Eq. (7)) derived in simplified LR treatment. This shows that the complete conformity is established only between the BC and simplified LR approaches since there is no collectivity except screening in either the BC or simplified version of LR treatment. The same result has been obtained for classical plasmas [16]. However for the later case the conformity is established only for the "smoothened" interaction potential, which decays faster than r^{-1} at large distances and increases slower than r^{-1} at small ones. Such a "smoothened" potential can be viewed as an alternative implementation of cutoffs. Due to the wave nature of the electrons the quantum mechanical description does not require a cutoff procedure at low distances and hence the conformity between BC and simplified LR approaches is valid for any interaction potential which decays faster than r^{-1}.

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Appendix: Some details for derivation of equation (31)

Here we give a more detail derivation of equation (31). When equation (30) is substituted into equation (28) and the interaction potential is written in **k**-space then $\mathbf{k}\cdot\mathbf{T}_{\alpha\beta}(\mathbf{k})$ term arises in equation (28), where

$$\mathbf{T}_{\alpha\beta}(\mathbf{k}) = \int d\mathbf{r}e^{-i\mathbf{kr}}\mathbf{P}_{\alpha\beta}(\mathbf{r}). \quad (A.1)$$

From the last expression we obtain

$$\mathbf{k}\cdot\mathbf{T}_{\alpha\beta}(\mathbf{k}) = -i\int d\mathbf{r}e^{-i\mathbf{kr}}\nabla\mathbf{P}_{\alpha\beta}(\mathbf{r}). \quad (A.2)$$

$\nabla\mathbf{P}_{\alpha\beta}(\mathbf{r})$ is calculated from equation (31)

$$\nabla\mathbf{P}_{\alpha\beta}(\mathbf{r}) = \frac{2m}{\hbar^2}(E_\alpha - E_\beta)\psi_\beta^{(0)}(\mathbf{r})\psi_\alpha^{(0)*}(\mathbf{r}), \quad (A.3)$$

which together with (A.2) and (16) immediately yield the following expression

$$\mathbf{k}\cdot\mathbf{T}_{\alpha\beta}(\mathbf{k}) = -\frac{2mi}{\hbar^2}(E_\alpha - E_\beta)S_{\beta\alpha}^*(\mathbf{k}). \quad (A.4)$$

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